Additional Modeling Problems

1. Consider the following displacement field for a material body:

$$u_1 = 0 \,, \qquad u_2 = 2X_1^2 \,, \qquad u_3 = 4X_2^2 \,.$$

   (a) Show that this deformation is possible in a continuously deformable body.

   (b) Determine the inverse mapping: $X = \varphi^{-1}(x, t)$.

2. A tensor Q is orthogonal if it preserves inner products

$$Qu \cdot Qv = u \cdot v \,,$$

   for vectors u, v. Prove that a *necessary and sufficient* condition that Q be orthogonal is:

$$Q^T = Q^{-1} \,.$$

3. Prove the following identities:

$$\nabla \times \nabla\phi = 0 \tag{1}$$
$$\nabla \cdot (\nabla \times v) = 0 \tag{2}$$