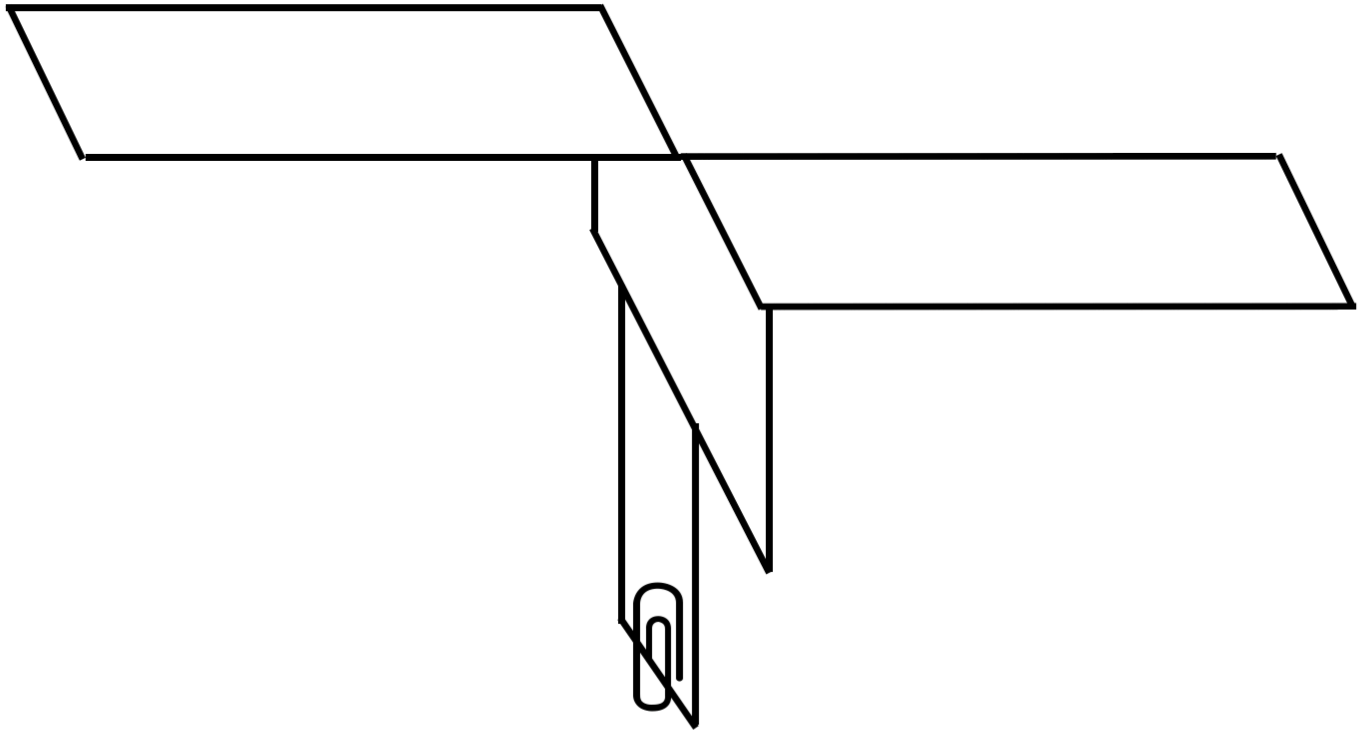
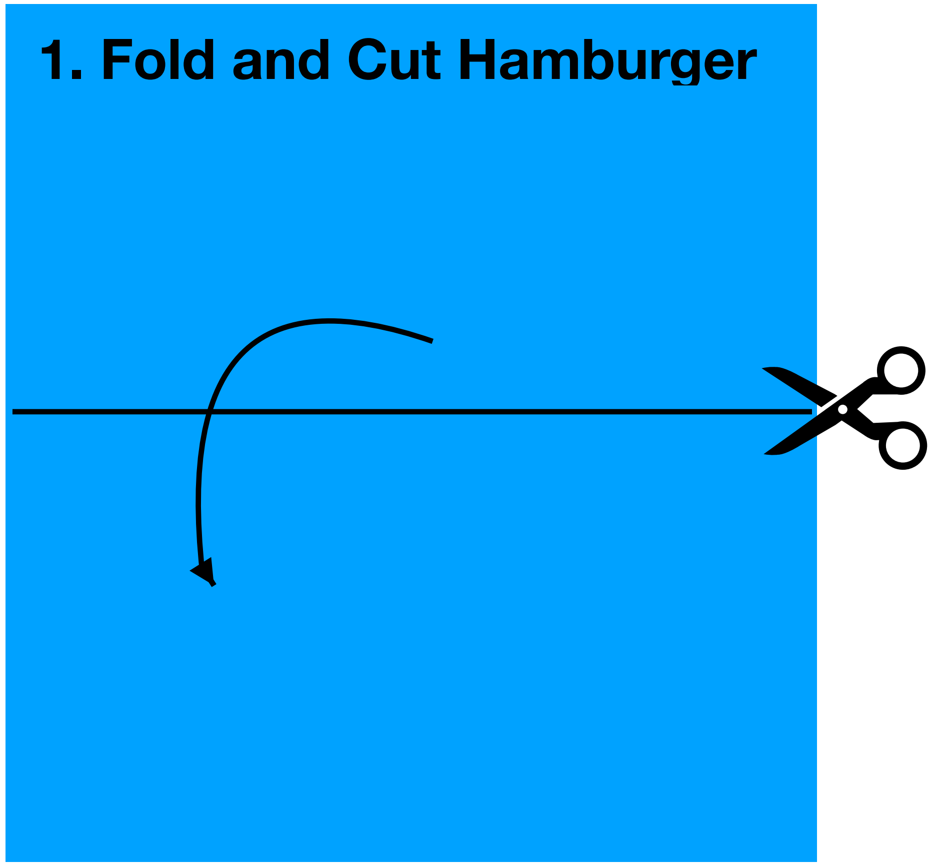
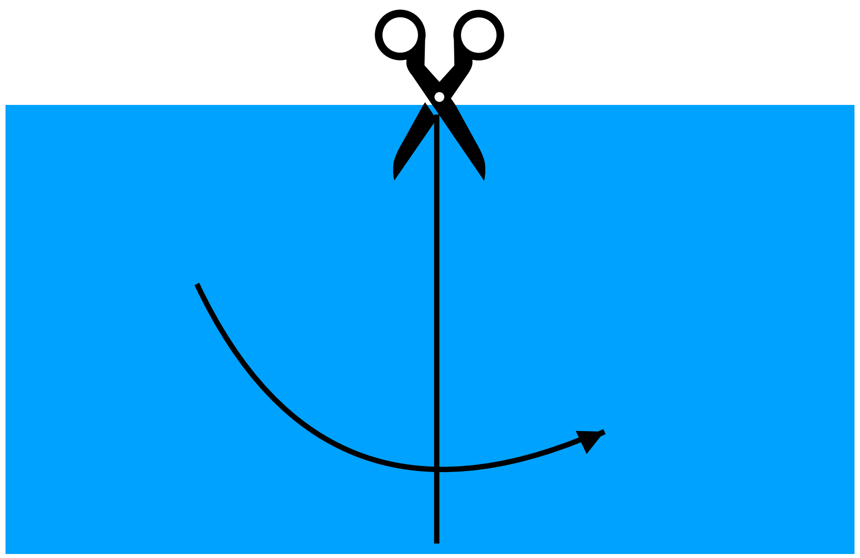
Dimensional Analysis for the Design of Models and Experiments



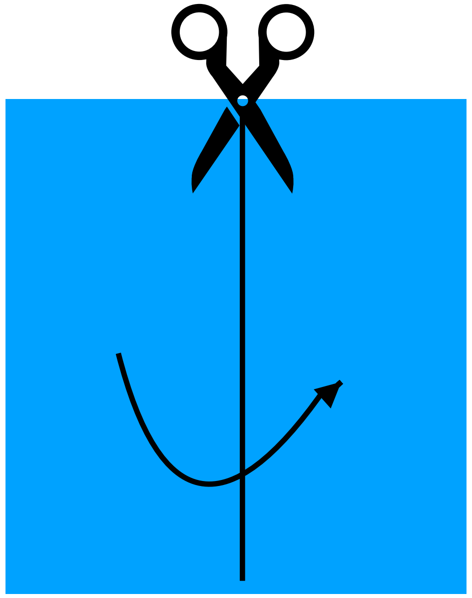
1. Fold and Cut Hamburger



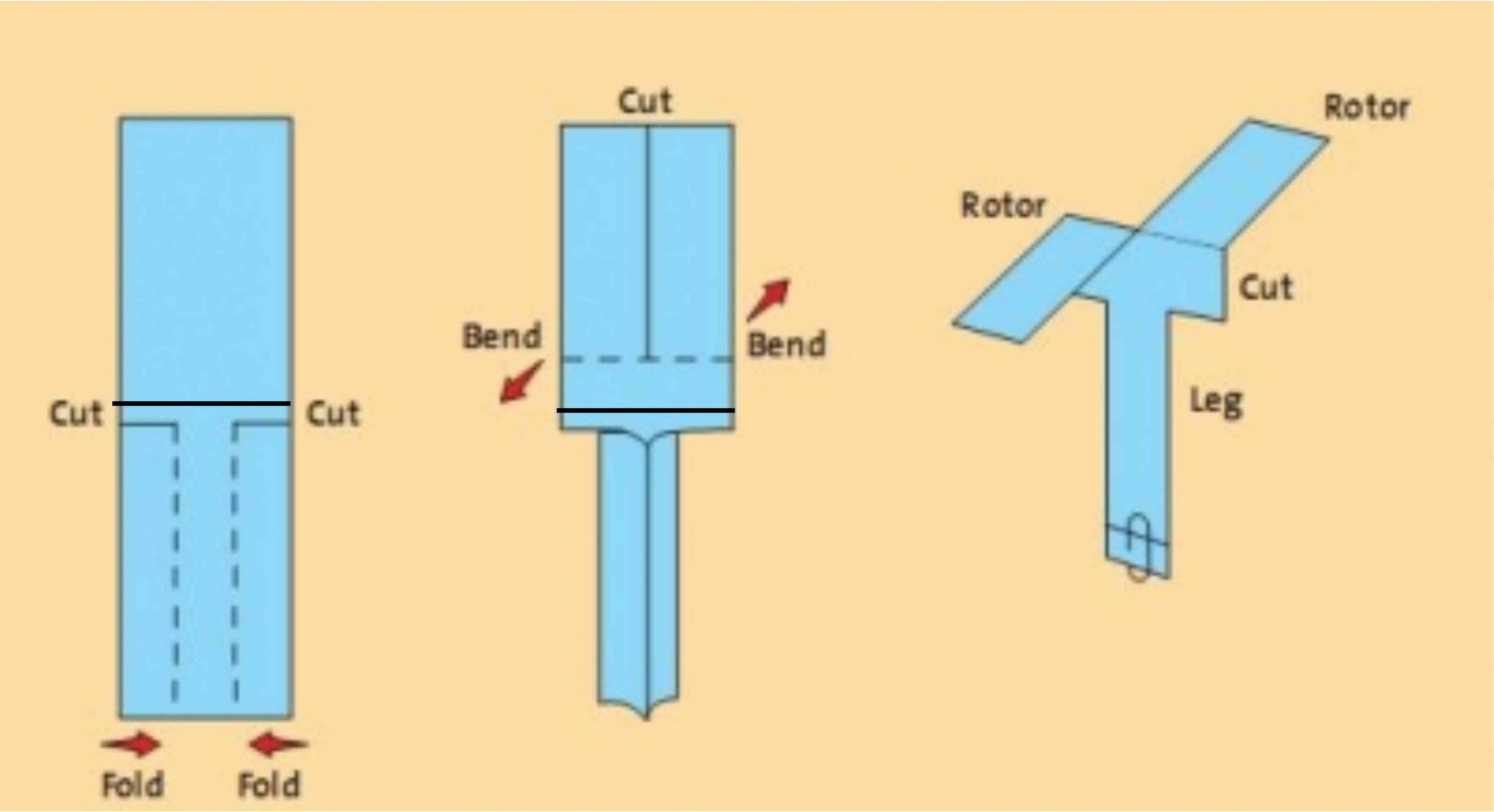
2. Fold and Cut Hamburger

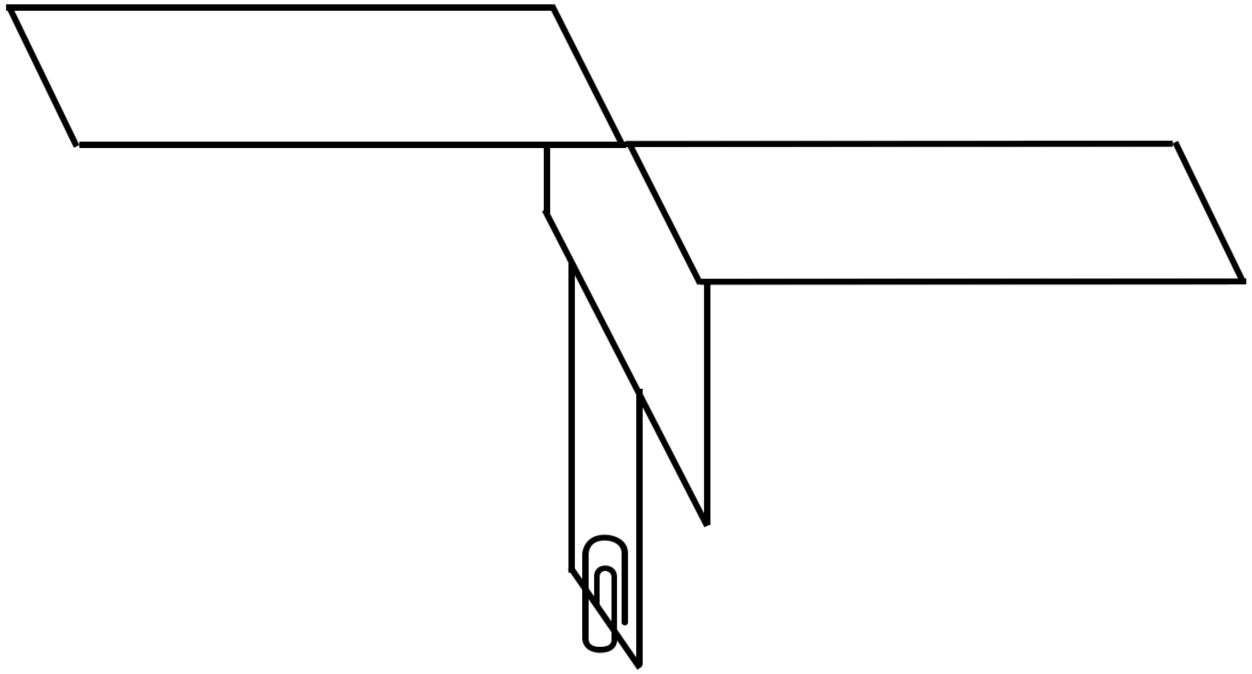


3. Fold and Cut Hotdog Style



4. Make Paper Helicopter



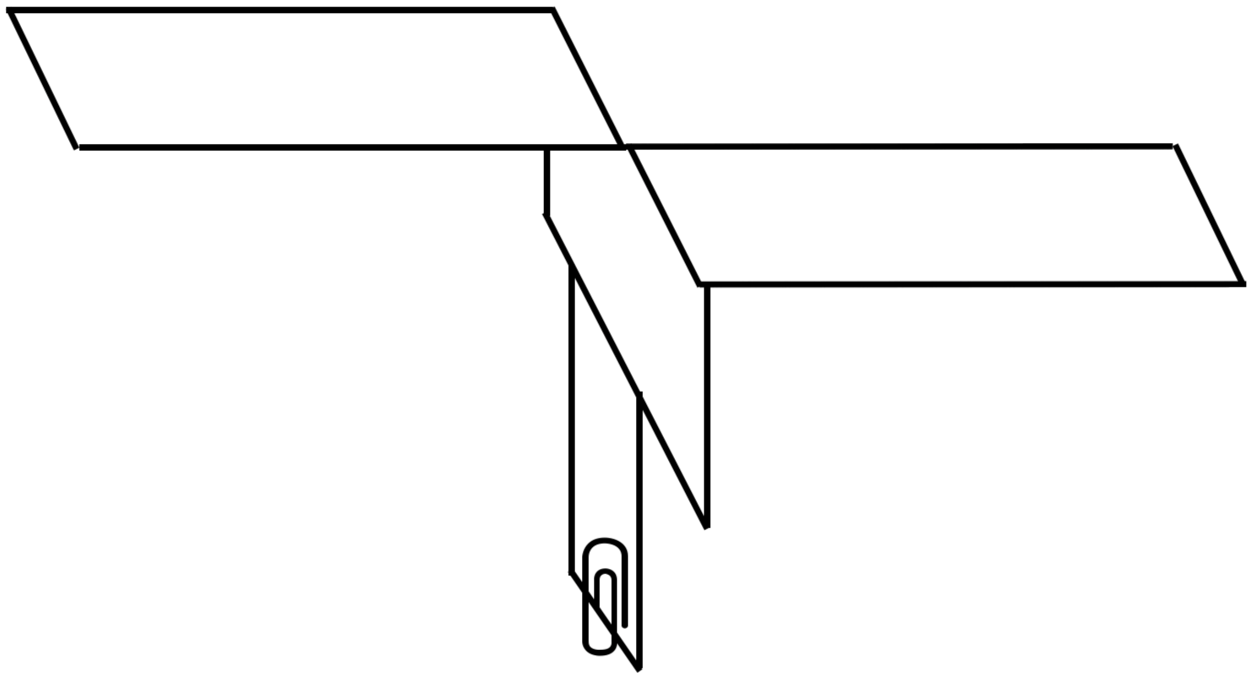


- We wish to model the flight time, T, for a paper helicopter dropped from a launch height H. In a group, discuss all of the possible variables T might depend on.

- **What are the consequences if you miss variables?**

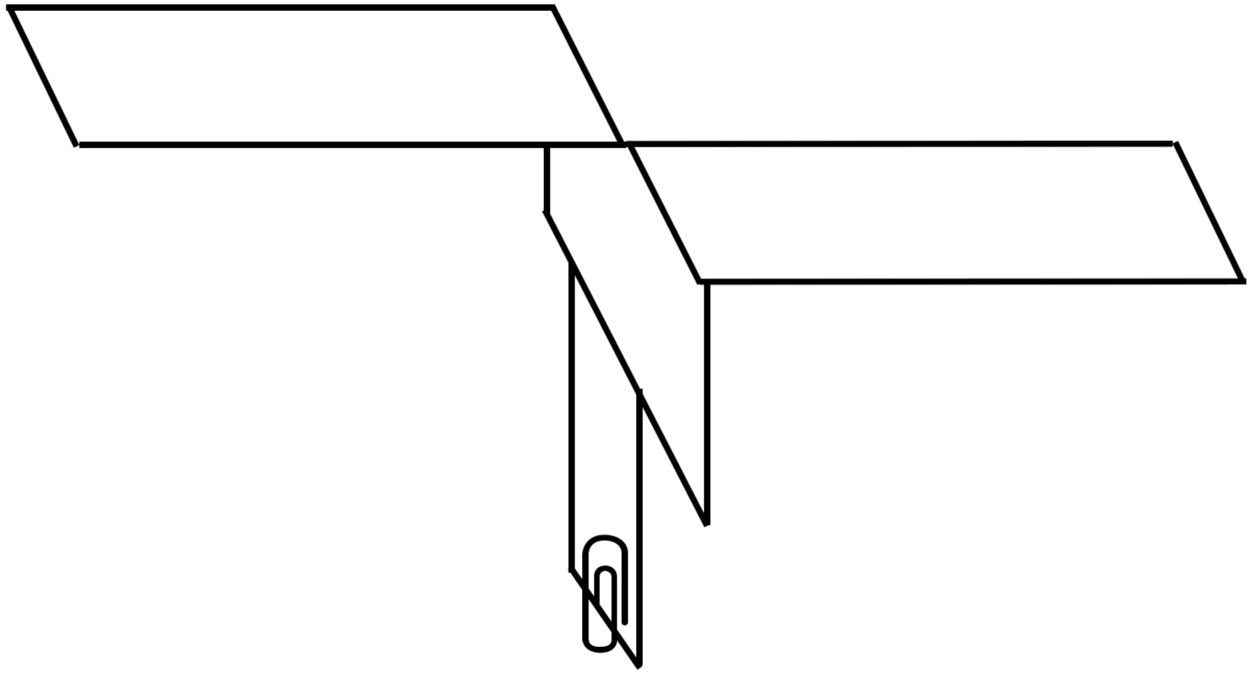
- **What are the consequences if you include too many variables?**

- **Is the *number* of variables you include important?**



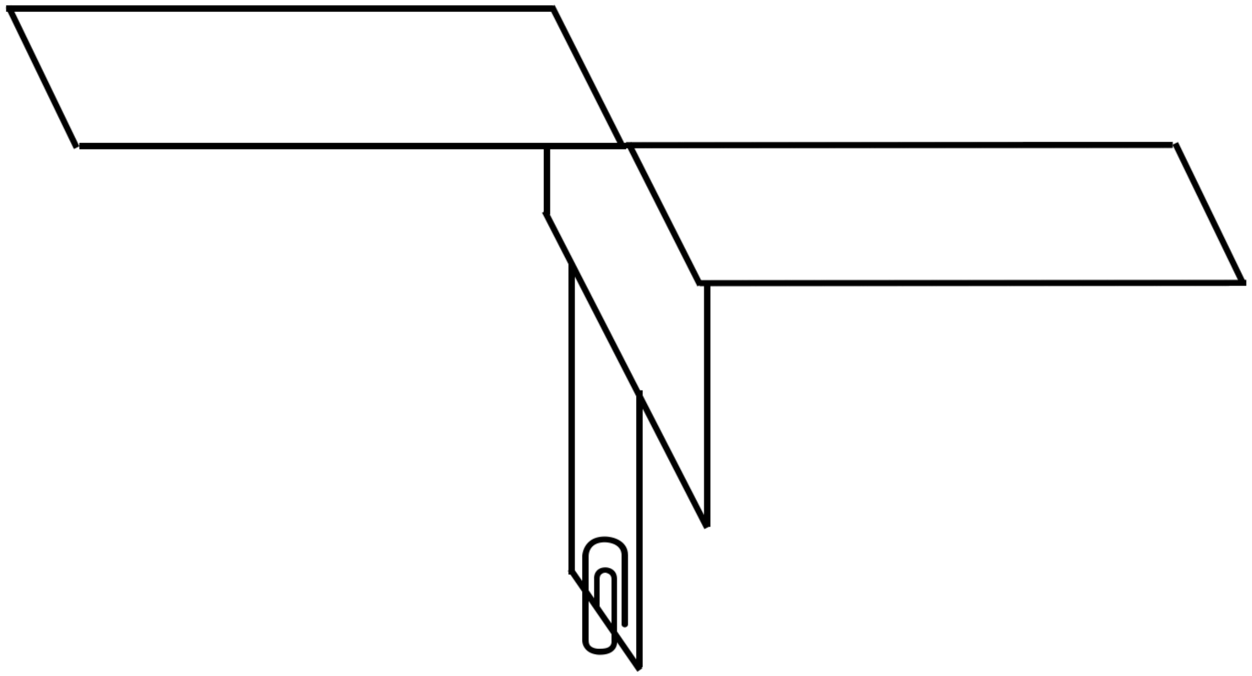
- Develop a dimensionless form for the relationship you identified in part 1. Identify “scaling variables” that you can use to represent the fundamental physical units.

- **What should you goals be when choosing scaling variables. Why?** *This is an important and strategic step in the process of model development.*



- Can you think of assumptions to simplify your model?

- **What types of things can be used to simplify models, in general?**



- Design/run an experiment to calibrate your model from part 3. Choose a sufficient launch height to allow for the helicopter to reach terminal velocity (~ 2m). The mass of the helicopter can easily be changed by adding paper clips. Run each experiment three times and average the flight times to help reduce noise.
- **How did dimensional analysis aid in the process of modeling and experimental design?**
- **What do your experiments reveal about the modeling assumptions made in part 2? Do you think these assumptions will hold for all parameter-regimes? What does this imply about your model?**

Variable	$\mathcal{T}$	H	ρ	g	r	m	ν	w	l	γ	θ
Unit	T	L	M/L^3	L/T^2	L	M	L^2/T	L	L	L	rad

T — Flight Time

H — Drop Height

ρ — Air Density

g — Gravity

r — Rotor radius

m — Mass

ν — Air Viscosity

w — Rotor Width

l — Tail Length

γ — Paper Thickness

θ — Angle of Rotors

$$\frac{H}{T} = v$$

Design ensures terminal velocity is reach *quickly* after being dropped

$$v = f\left(p, g, r, m, \nu, w, l, \gamma, \theta\right)$$

Variable	$\mathcal{T}$	H	ρ	g	r	m	ν	w	l	γ	θ
Unit	T	L	M/L^3	L/T^2	L	M	L^2/T	L	L	L	rad

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$$L_{ength} = r \qquad M_{ass} = \rho r^3$$

$$T_{ime} = \frac{r}{\sqrt{gr}}$$

$$v = f\left(p, g, r, m, \nu, w, l, \gamma, \theta\right)$$

$$\Downarrow$$

$$\frac{v}{\sqrt{gr}} = f\left(\frac{m}{\rho r^3}, \frac{\nu}{r\sqrt{gr}}, \frac{w}{r}, \frac{l}{r} . \frac{\gamma}{r}, \theta\right)$$

$$\Downarrow$$

$$\frac{H}{T\sqrt{gr}} = \frac{v}{\sqrt{gr}} = f\left(\frac{m}{\rho r^3}\right)$$

Flight Times (s)

$r/\#$clip	r=5cm	r=6cm	r=7cm
1	1.63	1.80	2.38
1	1.48	1.79	2.37
1	1.62	1.76	2.33
2	1.50	1.60	2.00
2	1.30	1.53	1.87
2	1.46	1.42	1.85
3	1.28	1.26	1.58
3	1.23	1.38	1.78
3	1.18	1.23	1.70

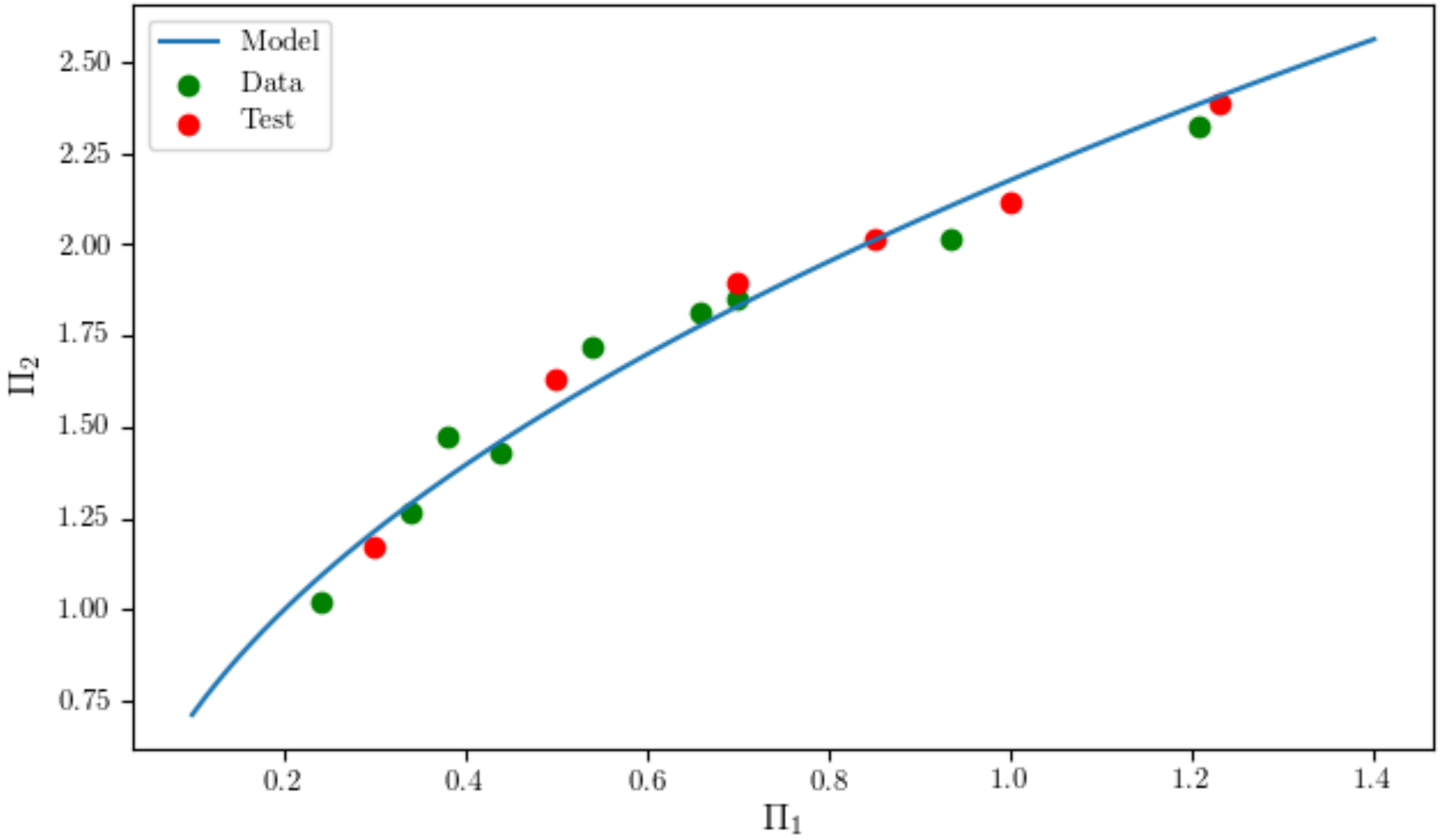
$$\frac{v}{\sqrt{gr}} \equiv \frac{H}{T\sqrt{gr}} = f\left(\frac{m}{\rho r^3}\right)$$

$$\Downarrow$$

$$\frac{H}{T\sqrt{gr}} = 2.18\left(\frac{m}{\rho r^3}\right)^{0.485}$$

Paper Helicopter Weight = 0.58g

Paper Clip Weight = 0.41g


Model
Data
Test
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
Π_2
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Π_1

Variable	$\mathcal{T}$	H	ρ	g	r	m	ν	w	l	γ	θ
Unit	T	L	M/L^3	L/T^2	L	M	L^2/T	L	L	L	rad

T — Flight Time

H — Drop Height

ρ — Air Density

ω — Weight (mg)

r — Rotor radius

ν — Air Viscosity

w — Rotor Width

l — Tail Length

γ — Paper Thickness

θ — Angle of Rotors

$$\frac{H}{T} = v$$

Design ensures terminal velocity is reach *quickly* after being dropped

$$v = f\left(p, \omega, r, \nu, w, l, \gamma, \theta\right)$$

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H — Drop Height

ρ — Air Density

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$$v = f\left(p, \omega, r, \nu, w, l, \gamma, \theta\right)$$

$$\Downarrow$$

$$\frac{vr\sqrt{\rho}}{\sqrt{\omega}} = f(\,\cdot\,)$$

$$\Downarrow$$

$$\frac{vr\sqrt{\rho}}{\sqrt{\omega}} = C$$

$$\parallel$$

$$\frac{H}{T} = C\sqrt{\left(\frac{\omega r^2}{\rho}\right)}$$

$$L_{ength} = r \qquad M_{ass} = \rho r^3$$

$$T_{ime} = \sqrt{\frac{\rho r^4}{\omega}}$$

Flight Times (s)

r/#clip	r=5cm	r=6cm	r=7cm
1	1.63	1.80	2.38
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$$\frac{H}{T} = 2.19\sqrt{\left(\frac{mgr^2}{\rho}\right)}$$

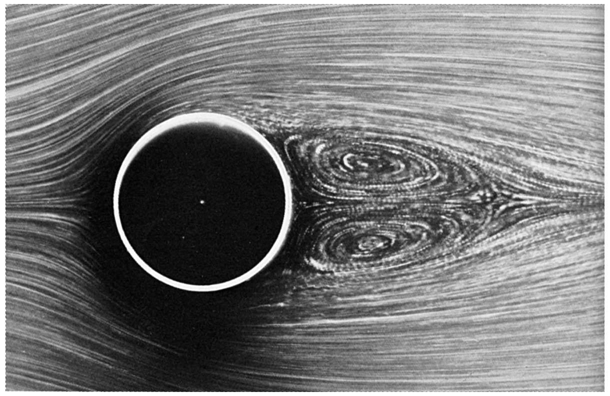
vs.

$$\frac{H}{T\sqrt{gr}} = 2.18\left(\frac{m}{\rho r^3}\right)^{0.485}$$

Paper Helicopter Weight = 0.58g

Paper Clip Weight = 0.41g

Drag over a cylinder



$$\frac{F}{L} = f(D, V, \rho, \mu, r)$$

$\frac{F}{L}$ — Drag Force

D — Diameter

V — Velocity

ρ — Density

μ — Viscosity

r — Roughness height

$$\frac{F/L}{\frac{1}{2}\rho D V^2} = f\left(\frac{\rho D V}{\mu}\right)$$

$$\Downarrow$$

$$\frac{F/L}{\frac{1}{2}\rho D V^2} = C_D$$

Drag over a cylinder

$$\frac{F/L}{\frac{1}{2}\rho D V^2} = f\left(\frac{\rho D V}{\mu}\right) \quad \Longrightarrow \quad \frac{F/L}{\frac{1}{2}\rho D V^2} = C_D$$