

## 2018-2019 Grand Challenge Award – Final Report

*Awardee:*       **Gregory J. Rodin, Professor,  
Aerospace Engineering and Engineering Mechanics**

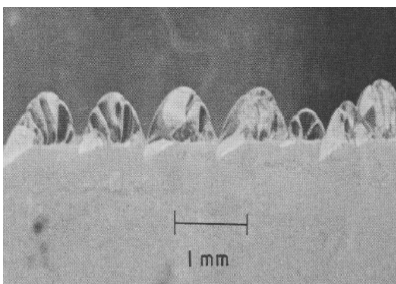


*Research Award Title:*       Modeling and Simulation of Three-Dimensional Crack Growth

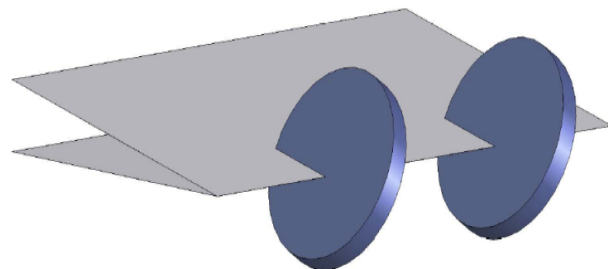
### Research Accomplishments

In the Fall semester of 2019, thanks to the Grand Challenge Award, I was able to focus almost exclusively on modeling three-dimensional crack growth. This problem is of central importance for many applications, and there is much remains to be understood.

There are two fundamentally different forms of three-dimensional crack growth. One is associated with sustained growth, and it is characterized by smooth crack fronts. The other is associated with initiation, and it is characterized by fragmented crack fronts, like that shown in Figure 1.



(a)



(b)

Figure 1. An initially straight crack front evolving into a fragmented one with secondary cracks (a); a schematic view of the primary and secondary cracks (courtesy of K. Ravi Chandran).

My research was primarily focused on smooth crack fronts. I proposed to view them as rods in the context of Eshelbian configurational mechanics, and, in that context, I developed a complete mechanical theory, including equilibrium, constitutive, and compatibility relationships. To this end, I heavily relied on classical differential geometry and the principle of virtual power, stated in the context of configurational rather than classical mechanics. Further, asymptotic analysis, based on my theory, revealed an intermediate length scale

$$\ell := \sqrt{\frac{r_p}{|\kappa_g|}}.$$

Here  $r_p$  is the size of the process zone ahead of the crack front, and  $\kappa_g$  is the geodesic curvature of the crack surface evaluated at the front. This intermediate length scale allows one to establish a non-ambiguous criterion for growth of cracks with smooth fronts. Results of my work have appeared in [1]. Let me mention that I have been working on this problem for two years, and, if it were not for the Grand Challenge Award, which allowed me to spend a long stretch of time at University of Minnesota, I probably would never be able to complete my work.

I also started on the more challenging problems of fragmented crack fronts. Again, using the framework of configurational mechanics, I was able to cast the problem as a competition between smooth crack growth versus fragmentation, or nucleation of secondary cracks as shown in Figure 1. This work is under development, but I am confident that it will come to fruition in the near future.

Finally, I completed my earlier work on Isogeometric Boundary Element Methods together with a former PhD student Matthias Taus, my colleague Tom Hughes, and a former post-doctoral student of Tom Michael Scott. In this work, we developed a family of Isogeometric Boundary Element Methods that allow one to carry out stress analysis of engineering components using the information directly available from CAD modeling. This opens realistic opportunities for developing fully automated design-through-analysis pipelines. This work appeared in [2].

## Publications

[1] Rodin, G.J., Local and nonlocal effects of three-dimensional crack growth initiation, *International Journal of Fracture* 221, 211-220 (2020).

[2] Taus, M., Rodin, G. J., Hughes, T.J.R., and Scott, M.A., Isogeometric Boundary Element Methods and Patch Tests for Linear Elastic Problems: Formulation, Numerical Integration, and Applications, *Computer Methods in Applied Mechanics and Engineering* 357 Article Number: UNSP 112591 Published: DEC 1 (2019).

## Awards

1. Visiting MTS Professorship at University of Minnesota, Fall 2019 was awarded by Department of Civil and Environmental Engineering at University of Minnesota to support my visit during the period covered by the Grand Challenge Award.

## Presentations

1. Rodin, G. J. Local and nonlocal aspects of three-dimensional crack growth, Invited talk at University of Minnesota, 2018.
2. Rodin, G. J. Local and nonlocal aspects of three-dimensional crack growth, SIAM regional meeting, Baton Rouge, LA, Fall 2018.
3. Rodin, G. J. Local and nonlocal aspects of three-dimensional crack growth, Invited talk at University of Berkeley, 2019.