ISOGEOMETRIC ANALYSIS-SUITABLE RECONSTRUCTION OF 1996 DODGE NEON USING RICCI FLOW WITH METRIC OPTIMIZATION

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ABSTRACT

Isogeometric analysis has gained significant traction over the last decade, but there still do not exist strong methods for reconstructing trimmed or faceted geometries into models suitable for boundary-fit spline analysis. In this paper, we integrate new theory and tools from computational geometry into a framework suitable for reconstructing industrially-relevant models. We demonstrate the efficacy of the framework by reconstructing the chassis of a 1996 Dodge Neon and using the reconstructed splines as input for isogeometric modal analysis in LS-DYNA.

Keywords: mesh generation, isogeometric analysis, computational geometry, quadrilateral, splines

1. INTRODUCTION

Isogeometric analysis [1] aims to integrate engineering design with engineering analysis by employing the same basis functions used in computer-aided design (CAD) for computer-aided engineering (CAE). Leveraging the smooth basis functions of CAD, isogeometric techniques give hope to streamline an otherwise labor-intesive design-through-anlaysis process [2] without approximating the intended design geometry, like traditional linear and bilinear meshing techniques. Most CAD models are constructed using trimmed B-spline and NURBS patches. Though trim-free tensor-product splines are inherently suitable for analysis, trimming operations introduce ambiguity between model geometry and topology, complicate integration, and prevent use of strong connectivity constraints or strong boundary condition imposition. Isogeometric models aiming to avoid these shortcomings need to be constructed from a set of trim-free boundary-aligned splines; to fit into the existing CAD framework, the splines must additionally be tensor-product NURBS patches. Such a model defined by a boundary-conforming set of NURBS patches is a semi-structured curvilinear quadrilateral mesh.

Generating a well-structured analysis-suitable quadrilateral mesh has been the subject of research for decades. Some of the most promising methods for computing these meshes on a trimmed model first compute a feature-aware triangulation of the surface (thus removing topological ambiguities from trimming), after which a quadrilateral mesh is extracted from the triangulation. Successful techniques for quad mesh computation include the frame field method, conformal methods, and techniques using Morse theory. However, to be useful in isogeometric engineering analysis, the quadrilateral mesh generation process must meet the following quality criteria:

- Quickly and reliably produces all-quad meshes
- Singular points (interior vertices without four edges emanating from them or boundary points without three edges emanating from them) connect with other singular points in short paths without unnecessary winding
- Meshes align to features

- Computational techniques easily integrate expert-user information

While conformal and Morse theoretic methods come with theoretical proofs of reliability, they generally cannot align to features of engineering surfaces. Conversely, frame field methods are fast and feature-aligned, but often fail to produce all-quadrilateral meshes. Furthermore, many frame field methods cannot easily incorporate user input.

In this work, we combine the strengths of conformal techniques with nonlinear parameterization techniques to dependably generate feature-aware meshes with good connectivity between singular points that can easily integrate user-defined constraints. We show the potential of this method by reconstructing the entire body-in-white of a 1996 Dodge Neon [3] from an unstructured quadrilateral mesh with some triangles into a set of smooth, boundary and feature-aligned isogeometric analysis-suitable bicubic NURBS patches.

2. THEORETICAL FOUNDATIONS

A curvilinear quadrilateral mesh on a surface is equivalent to a quad mesh metric [4], defined as follows:

Definition 2.1. *A quad mesh metric on surface S is a Riemannian metric $\langle\cdot,\cdot\rangle_Q$ with a finite set of cone singularities P and an ancillary cross field, C_Q, obtained by parallel transport under $\langle\cdot,\cdot\rangle_Q$ of a unit cross from $p \in S-P$ to all of $S-P$, with the following properties:*

P1 *$\langle\cdot,\cdot\rangle_Q$ is a flat metric, and the total curvature measure of the singularities obeys the following generalized Gauss-Bonnet condition:*

$$\sum_{q\in P\cap\partial S}\big(\pi-\theta(q)\big)+\sum_{p\in P-\partial S}\big(2\pi-\theta(p)\big)=2\pi\chi(S) \tag{1}$$

where $\chi(S)$ is the surface Euler characteristic, $\theta(p)$ is an interior point p's contribution to the Gaussian curvature measure under $\langle\cdot,\cdot\rangle_Q$, and $\theta(q)$ is a boundary point q's contribution to the geodesic curvature measure under $\langle\cdot,\cdot\rangle_Q$.

P2 *The holonomy group of $S-P$ under the Levi-Cevita connection of metric $\langle\cdot,\cdot\rangle_Q$ is a subgroup of $\mathcal{R}=\{\exp(i\frac{k\pi}{2}, k\in\mathbb{Z}\}$.*

P3 *C_Q is boundary-aligned, meaning that an axis of C_Q is locally parallel to ∂S everywhere.*

P4 *Integral curves of C_Q are geodesics of $\langle\cdot,\cdot\rangle_Q$.*

P5 *Integral curves of C_Q are of finite length.*

For a feature-aligned mesh, the above cross field must also align to features in a manner similar to **P3**.

3. COMPUTATIONAL FRAMEWORK

Computationally, a quad mesh metric is represented as a special immersion of a cut version of a surface into the plane [5, 6, 7]. Here, we use this fact but motivate using the quad mesh metric above.

To compute a feature-aligned quadrilateral mesh for a surface, we take the following three-step approach:

1. Define cone singularities on the mesh (e.g. automatically using [8, 9]),
2. Compute discrete surface Ricci flow [10],
3. Optimize the computed metric for boundary alignment, feature alignment, and better singularity connectivity using [7, 11].

Figure 1 presents a pictorial depiction of this process on a beam of the DEVCOM Generic Hull vehicle [12].

Step 1 is used to establish locations of cone singularities satisfying Equation 1 of **P1** and Property **P2**; the second step produces a flat metric satisfying Properties **P1**, **P2**, and **P4**. If cone singularities are placed well,[1] the computed metric will nearly satisfy Property **P3**. As a result, the constrained inversion-precluding optimization of Step 3 will ensure that the metric remains valid and flat, while penalty constraints enforce the remaining properties. For open shells, connectivity constraints are semi-automatically deduced using homotopy theory [13].

4. RESULTS

Figure 1 demonstrates this framework for semi-automatically computing a trim-free spline reconstruction of a DEVCOM Generic Hull vehicle beam. The robustness of the method is demonstrated by rebuilding the body-in-white of a finite element mesh of a 1996 Dodge Neon [3], shown in Figure 2. Analysis-suitability is shown through isogometric modal analysis of parts in LS-DYNA (see Figure 3).

5. CONCLUSION AND FUTURE WORK

Herein, we presented a theory-based computational framework that defines an isogeometric analysis-suitable quadrilateral parameterization on a surface. The methods are shown to be able to rebuild trimmed and faceted geometries, and culminate by rebuilding the chassis of a 1996 Dodge Neon [3] as a set of conforming bicubic NURBS patches.

While this theory is well-defined, the current implementation needs further development for commercial use (including better code implementation, GUI, etc). Future work will involve performing an isogeometric crash analysis of the Neon body-in-white. Additionally, further work will involve a refinement study of

[1] Our experience shows that cones computed by minimizing a Dirichlet-type energy for a frame field as in [8, 9] are often well-placed.

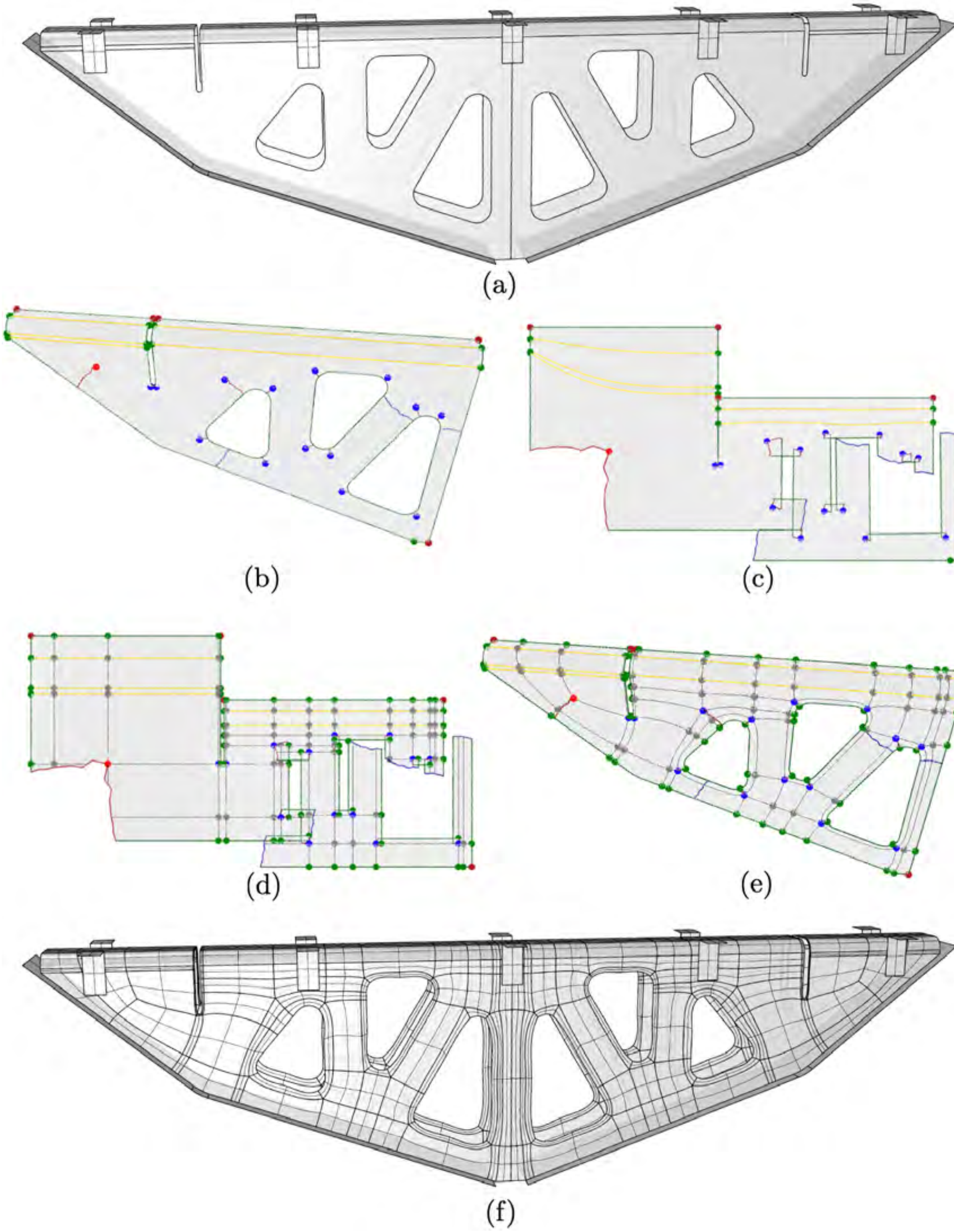

Figure 1: The symmetric portion of the trimmed DEVCOM Generic Hull vehicle shown in (a) is converted into a feature-aware triangulation, whereupon cone singularities are computed in (b). Here, points in red are cone singularities with valence less than four, points in blue are cones with valence greater than 4, points in green are feature points, points in grey are normal vertices of the resulting quad layout, curves in gold are feature curves, and green curves are surface boundaries. Discrete surface Ricci flow is then computed on the surface, and the surface is then cut to a topological disk with additional cuts to cone singularities and then immersed into the plane in (c). Cuts to cone singularities are shown in red, while cuts between boundary components are shown in blue. Using a metric-preserving nonlinear optimization, the immersion is modified into an immersion in (d) that induces the quadrilateral layout on the surface shown in (e). Here, black lines in (d) correspond to the curvilinear lines in (e). Using this layout, the original trimmed surface of (a) is reconstructed into the set of bicubic splines shown in (f).

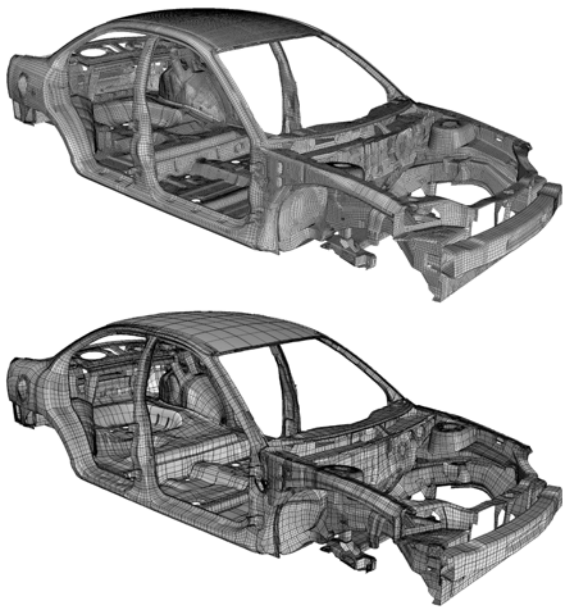

Figure 2: The unstructured finite element Dodge Neon mesh of [3] (above) is converted to a semi-structured bicubic set of NURBS splines (below).

the isogometric spline spaces compared to traditional mesh elements to evaluate the accuracy of each.

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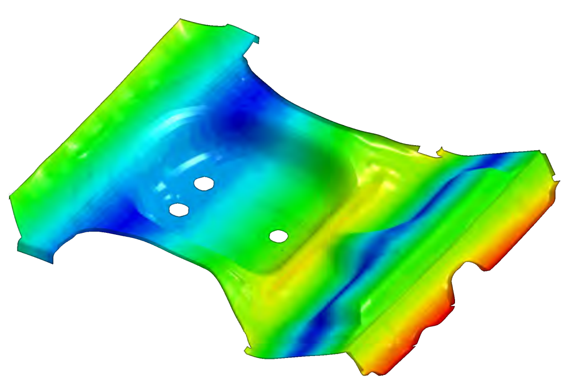

Figure 3: The seventh mode of a reconstructed rear car floorboard using bicubic NURBS patches is shown.